

## SHORTER COMMUNICATIONS

### A NOTE ON THE LOCKHART-MARTINELLI CORRELATION IN THE TURBULENT-TURBULENT CASE

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#### NOMENCLATURE

|            |   |
|------------|---|
| $b$ ,      | Blasius exponent;   |
| $K$ ,      | dimensionless parameter, equation (5);                          |
| $m$ ,      | dimensionless parameter to be fitted to the experimental data;  |
| $X_{TT}$ , | Lockhart-Martinelli parameter for the turbulent-turbulent case. |

#### Greek symbols

|            |  |
|------------|--|
| $\alpha$ , | void fraction;   |
| $\gamma$ , | slip ratio;  |
| $\mu_g$ ,  | vapor viscosity;   |
| $\mu_l$ ,  | liquid viscosity;  |
| $\rho_g$ , | vapor density;   |
| $\rho_l$ , | liquid density;  |
| $\phi_L$ , | parameter related to the void fraction $\alpha$ by equation (2); |
| $\chi$ ,   | vapor quality.   |

FOR TWO phase turbulent-turbulent flow in pipes, the void fraction  $\alpha$  depends on the vapor quality,  $\chi$ , on the vapor and liquid densities  $\rho_g, \rho_l$  on the vapor and liquid viscosities  $\mu_g, \mu_l$ . That  $\alpha$  actually only depends on the parameter  $X_{TT}$  is the physical content of the Lockhart-Martinelli correlation [1],  $X_{TT}$  being defined by

$$X_{TT} = [(1 - \chi)/\chi]^{(2-b)/2} (\rho_g/\rho_l)^{1/2} (\mu_l/\mu_g)^{b/2} \quad (1)$$

where  $b$  denotes the Blasius exponent.

The curve  $\alpha(X_{TT})$  is usually fitted by the equations

$$\alpha = 1 - (1/\phi_L), \quad (2)$$

$$\phi_L^2 = 1 + 21/X_{TT} + 1/X_{TT}^2.$$

Using such a fit is rather troublesome when performing analytical calculations. Moreover equation (2) yields an unphysical singularity for  $\partial\alpha/\partial\chi$  as  $\chi \rightarrow 0$ . As a substitute for equation (2) we propose the relation

$$\alpha = 1 - (m/2) X_{TT}^{2(2-b)} - \{[1 + (m/2) X_{TT}^{2(2-b)}]^2 - 1\}^{1/2} \quad (3)$$

where  $m$  denotes a dimensionless parameter to be fitted to the experimental data.

A comparison of the fit of equations (2) and (3) is given in Table 1, with  $m = 0.054$ . The agreement is quite good, except for  $\chi \rightarrow 0$ , since equation (3) does not generate any unphysical singularity.

$\alpha(\chi)$ , as defined by equation (3), only depends on  $X_{TT}$ , according to the Lockhart-Martinelli correlation. This is the only case of equation (3) which can be turned into the simple analytical relation

$$\chi = (zmK)/[(1 - \alpha)^2 + \alpha mK] \quad (4)$$

where  $K$  is defined by

$$K = [(\rho_g/\rho_l)^{1/2} (\mu_l/\mu_g)^{b/2}]^{2/(2-b)}. \quad (5)$$

Let us now consider the slip ratio  $\gamma$ , which is known to satisfy the identity

$$\gamma = (\rho_l/\rho_g) [(1 - \alpha)/\alpha] [\chi/(1 - \chi)]. \quad (6)$$

Using equation (4), this simplifies to

$$\gamma = (\rho_l/\rho_g) [mK/(1 - \alpha)]. \quad (7)$$

Some properties of the correlations, equations (4) and (7) are:

- (1) From the numerical point of view, the agreement with the usual correlation, equation (2), is quite good.
- (2) They do not yield unphysical singularities as  $\chi \rightarrow 0$ .
- (3) Their analytical form is very simple.
- (4) They only depend on the parameter  $m$ , which must be

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Table 1. Comparison of the correlations equations (2) and (3)

| $X_{TT}$ | $\chi$             | $\alpha$ [equation (2)] | $\alpha$ [equation (3)] |
|----------|--------------------|-------------------------|-------------------------|
| 55.21    | $2 \times 10^{-4}$ | 0.149                   | 0.137                   |
| 30.20    | $4 \times 10^{-4}$ | 0.232                   | 0.223                   |
| 21.22    | $6 \times 10^{-4}$ | 0.291                   | 0.284                   |
| 13.52    | $8 \times 10^{-4}$ | 0.337                   | 0.330                   |
| 16.60    | $1 \times 10^{-3}$ | 0.374                   | 0.368                   |
| 7.436    | $2 \times 10^{-3}$ | 0.490                   | 0.487                   |
| 5.221    | $3 \times 10^{-3}$ | 0.555                   | 0.553                   |
| 4.062    | $4 \times 10^{-3}$ | 0.599                   | 0.598                   |
| 3.342    | $5 \times 10^{-3}$ | 0.632                   | 0.631                   |
| 2.849    | $6 \times 10^{-3}$ | 0.657                   | 0.656                   |
| 2.489    | $7 \times 10^{-3}$ | 0.677                   | 0.677                   |
| 2.214    | $8 \times 10^{-3}$ | 0.694                   | 0.694                   |
| 1.997    | $9 \times 10^{-3}$ | 0.708                   | 0.709                   |
| 1.821    | $1 \times 10^{-2}$ | 0.721                   | 0.721                   |
| 0.9873   | $2 \times 10^{-2}$ | 0.793                   | 0.794                   |
| 0.6877   | $3 \times 10^{-2}$ | 0.828                   | 0.829                   |
| 0.5306   | $4 \times 10^{-2}$ | 0.849                   | 0.851                   |
| 0.4330   | $5 \times 10^{-2}$ | 0.865                   | 0.866                   |
| 0.3661   | $6 \times 10^{-2}$ | 0.877                   | 0.878                   |
| 0.3172   | $7 \times 10^{-2}$ | 0.886                   | 0.887                   |
| 0.2798   | $8 \times 10^{-2}$ | 0.894                   | 0.894                   |
| 0.2501   | $9 \times 10^{-2}$ | 0.900                   | 0.901                   |
| 0.2260   | $1 \times 10^{-1}$ | 0.906                   | 0.906                   |
| 0.08691  | 0.25               | 0.948                   | 0.945                   |
| 0.03342  | 0.50               | 0.974                   | 0.968                   |
| 0        | 1                  | 1                       | 1                       |

In order to relate the vapor quality  $\chi$  and the parameter  $X_{TT}$  the following parameters have been used (corresponding to sodium):  $\rho_l = 0.7217 \text{ g cm}^{-3}$ ,  $\rho_g = 0.48 \cdot 10^{-3} \text{ g cm}^{-3}$ ,  $\mu_l = 0.1475 \text{ centipoises}$ ,  $\mu_g = 0.02009 \text{ centipoises}$ ,  $b = 0.26$ . In equation (3) the parameter  $m$  has been fitted to  $m = 0.054$ .

fitted to the experimental results. The usual correlation, equation (2), could be adjusted either by modifying the coefficients of the series in  $1/X_{TT}$  or by adding to it further terms.

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#### REFERENCES

1. R. W. Lockhart and R. C. Martinelli, *Chem. Engng Prog.* 45, 39 (1949).